

Charged lepton and Neutrino masses from a low energy $SU(3)$ flavour symmetry model

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We report a recent study on lepton masses within a beyond standard model with a $SU(3)$ family symmetry model. In this scenario ordinary heavy fermions, top and bottom quarks and tau lepton become massive at tree level from **Dirac See-saw** mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions U, D, E, N , with N a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general tree level See-saw Majorana neutrino mass matrix with four massless eigenvalues. Hence, light fermions, including light neutrinos, obtain masses from one loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. Recent analysis shows the existence of a space parameter region where the M_U, M_D, M_E, M_N vector-like fermion masses are within a scale of a few TeV's. This BSM model is able to accommodate the known spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM} as well as the charged lepton masses. We report a preliminary solution for ordinary charged lepton masses $(m_e, m_\mu, m_\tau) = (0.486, 102.7, 1746.17)$ MeV at the M_Z scale, a $M_E \approx 2.6$ TeV and $SU(3)$ gauge boson masses of $\mathcal{O}(10 \text{ TeV})$.

A detailed numerical analysis on neutrino masses and mixing is in progress and hopefully shall be reported soon.

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I. INTRODUCTION

Recently several experiments have reported new experimental results on neutrino mixing[1], on large θ_{13} mixing from Daya Bay[2], T2K[3], MINOS[4], DOUBLE CHOOZ[5], and RENO[6], implying a deviation from TBM[26] scenario, including possible evidence for the existence of sterile neutrinos from LSND and MiniBooNE[7, 8].

The strong hierarchy of quark and charged lepton masses and quark mixing have suggested to many model building theorists that light fermion masses could be generated from radiative corrections[11], while those of the top and bottom quarks as well as that of the tau lepton are generated at tree level. This may be understood as a consequence of the breaking of a symmetry among families (a horizontal symmetry). This symmetry may be discrete [12], or continuous, [13]. The radiative generation of the light fermions may be mediated by scalar particles as it is proposed, for instance, in references [14, 15] and the author in [27], or also through vectorial bosons as it happens for instance in "Dynamical Symmetry Breaking" (DSB) and theories like " Extended Technicolor " [16].

In this report we address the problem of fermion masses and quark mixing within an extension of the SM introduced by the author in [17], which includes a $SU(3)$ [18] gauged flavor symmetry commuting with the SM group. In previous reports[19] we showed that this model has the properties to accommodate a realistic spectrum of charged fermion masses and quark mixing. We introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[20] mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions. Recently, some authors have pointed out interesting features regarding the possibility of the existence of a sequential fourth generation[21]. Theories and models with extra matter may also provide interesting scenarios for present cosmological problems, such as candidates for the nature of the Dark Matter ([22],[23]). This is the case of an extra generation of vector-like matter, both from theory and current experiments[24]. Due to the fact that the vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks yield an extended 4×4 non-unitary CKM quark mixing matrix. It has pointed out for some authors that these type of vector-like fermions are weakly constrained from Electroweak Precision Data (EWP) because they do not break directly the custodial symmetry, then main experimental constraints on vector-like matter come from the direct production bounds and their implications on flavor physics. See ref. [24] for further details on constraints for $SU(2)_L$ singlet vector-like fermions.

II. MODEL WITH $SU(3)$ FLAVOR SYMMETRY

A. Fermion content

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where $SU(3)$ is a flavor symmetry among families and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with g_s , g and g' the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

$$\psi_q^o = (3, 3, 2, \frac{1}{3})_L \quad , \quad \psi_u^o = (3, 3, 1, \frac{4}{3})_R \quad , \quad \psi_d^o = (3, 3, 1, -\frac{2}{3})_R$$

$$\psi_l^o = (3, 1, 2, -1)_L \quad , \quad \psi_e^o = (3, 1, 1, -2)_R \quad ,$$

where the last entry corresponds to the hypercharge Y , and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions: Right handed neutrinos $\Psi_\nu^o = (3, 1, 1, 0)_R$, and the $SU(2)_L$ singlet vector-like fermions

$$U_{L,R}^o = (1, 3, 1, \frac{4}{3}) \quad , \quad D_{L,R}^o = (1, 3, 1, -\frac{2}{3}) \quad (1)$$

$$N_{L,R}^o = (1, 1, 1, 0) \quad , \quad E_{L,R}^o = (1, 1, 1, -2) \quad (2)$$

The above fermion content and its assignment under the group G make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the canonical form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[25]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses together with the radiative corrections.

III. ELECTROWEAK SYMMETRY BREAKING

Recently ATLAS[9] and CMS[10] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to an BSM theory. The electroweak symmetry breaking in the $SU(3)$ family symmetry model involve the introduction of two triplets of $SU(2)_L$ Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to $U(1)_Q$, we introduce the scalars: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the VEVs: $\langle \Phi^u \rangle^T = (\langle \Phi_1^u \rangle, \langle \Phi_2^u \rangle, \langle \Phi_3^u \rangle)$, $\langle \Phi^d \rangle^T = (\langle \Phi_1^d \rangle, \langle \Phi_2^d \rangle, \langle \Phi_3^d \rangle)$, where T means transpose, and

$$\langle \Phi_i^u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_i \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_i^d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V_i \end{pmatrix} . \quad (3)$$

The contributions from $\langle \Phi^u \rangle$ and $\langle \Phi^d \rangle$ yield to the W and Z gauge boson masses

$$\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z_o^2 \quad (4)$$

$v_u^2 = v_1^2 + v_2^2 + v_3^2$, $v_d^2 = V_1^2 + V_2^2 + V_3^2$. Hence, if we define as usual $M_W = \frac{1}{2}gv$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

IV. $SU(3)$ FLAVOR SYMMETRY BREAKING

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)$, we introduce the flavon scalar fields: η_i , $i = 2, 3$, transforming under the gauge group as $(3, 1, 1, 0)$ and taking the "Vacuum Expectation Values" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \quad (5)$$

The above scalar fields and VEV's break completely the $SU(3)$ flavor symmetry. The corresponding $SU(3)$ gauge bosons are defined in Eq.(15) through their couplings to fermions. Thus, a natural hierarchy among the VEVs scales is $\Lambda_2, \Lambda_3 \gg v \simeq 246$ GeV. Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain in good approximation the gauge bosons mass terms

$$\begin{aligned} & \frac{M_1^2}{2} Z_1^2 + \left(\frac{4}{3} M_2^2 + \frac{1}{3} M_1^2 \right) \frac{Z_2^2}{2} - \frac{M_1^2}{\sqrt{3}} Z_1 Z_2 \\ & M_1^2 Y_1^+ Y_1^- + M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^- \end{aligned} \quad (6)$$

$$M_1^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2}, \quad M_2^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2}, \quad M_3^2 = M_1^2 + M_2^2 \quad (7)$$

The diagonalization of the $Z_1 - Z_2$ squared mass matrix yields the eigenvalues

$$\begin{aligned} M_-^2 &= \frac{2}{3} \left(M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right) \\ M_+^2 &= \frac{2}{3} \left(M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right) \end{aligned} \quad (8)$$

and therefore the gauge boson masses

$$M_-^2 \frac{Z_-^2}{2} + M_+^2 \frac{Z_+^2}{2} + M_1^2 Y_1^+ Y_1^- + M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^- \quad (9)$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \quad (10)$$

$$\cos \phi \sin \phi = \frac{\sqrt{3}}{4} \frac{M_1^2}{\sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}}$$

with the hierarchy $M_1, M_2, M_-, M_+ \gg M_W$.

V. FERMION MASSES

A. Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings

$$h \bar{\psi}_l^o \Phi^d E_R^o + h_2 \bar{\psi}_e^o \eta_2 E_L^o + h_3 \bar{\psi}_e^o \eta_3 E_L^o + M \bar{E}_L^o E_R^o + h.c \quad (11)$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_1, h_2 and h_3 are Yukawa coupling constants.

When the involved scalar fields acquire VEV's we get, in the gauge basis $\psi_{L,R}^o{}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h v_1 \\ 0 & 0 & 0 & h v_2 \\ 0 & 0 & 0 & h v_3 \\ 0 & h_2 \Lambda_2 & h_3 \Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \quad (12)$$

Notice that \mathcal{M}^o has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call \mathcal{M}^o a **"Dirac See-saw"** mass matrix. \mathcal{M}^o is diagonalized by applying a biunitary transformation $\psi_{L,R}^o = V_{L,R}^o \chi_{L,R}$. The orthogonal matrices V_L^o and V_R^o are obtained explicitly in the Appendix A. From V_L^o and V_R^o , and using the relationships defined in this Appendix, one computes

$$V_L^{oT} \mathcal{M}^o V_R^o = \text{Diag}(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) \quad (13)$$

$$V_L^{oT} \mathcal{M}^o \mathcal{M}^{oT} V_L^o = V_R^{oT} \mathcal{M}^{oT} \mathcal{M}^o V_R^o = \text{Diag}(0, 0, \lambda_-, \lambda_+). \quad (14)$$

where λ_- and λ_+ are the nonzero eigenvalues defined in Eqs.(A4-A5), $\sqrt{\lambda_+}$ being the fourth heavy fermion mass, and $\sqrt{\lambda_-}$ of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(13,14) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

B. One loop contribution to fermion masses

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 1. Internal fermion line in this diagram represent the Dirac see-saw mechanism implemented by the couplings in Eq.(11). The vertices read from the $SU(3)$ flavor symmetry interaction Lagrangian

$$i\mathcal{L}_{int} = \frac{g_H}{2} (\bar{e}^o \gamma_\mu e^o - \bar{\mu}^o \gamma_\mu \mu^o) Z_1^\mu + \frac{g_H}{2\sqrt{3}} (\bar{e}^o \gamma_\mu e^o + \bar{\mu}^o \gamma_\mu \mu^o - 2\bar{\tau}^o \gamma_\mu \tau^o) Z_2^\mu \\ + \frac{g_H}{\sqrt{2}} (\bar{e}^o \gamma_\mu \mu^o Y_1^+ + \bar{e}^o \gamma_\mu \tau^o Y_2^+ + \bar{\mu}^o \gamma_\mu \tau^o Y_3^+ + h.c.) , \quad (15)$$

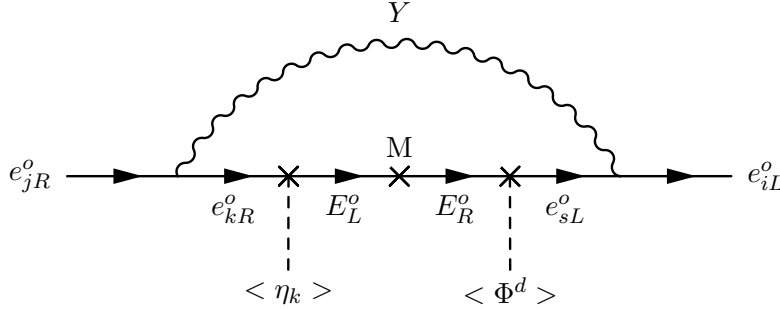


FIG. 1: Generic one loop diagram contribution to the mass term $m_{ij} \bar{e}_{iL}^o e_{jR}^o$.

where g_H is the $SU(3)$ coupling constant, Z_1 , Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(11) after the scalar fields get VEV's. The one loop diagram of Fig.1 gives the generic contribution to the mass term $m_{ij} \bar{e}_{iL}^o e_{jR}^o$

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \quad , \quad \alpha_H \equiv \frac{g_H^2}{4\pi} \quad (16)$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(15), $m_3^o = -\sqrt{\lambda_-}$ and $m_4^o = \sqrt{\lambda_+}$ are the See-saw mass eigenvalues, Eq.(13), and $f(x, y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix A, we compute

$$\sum_{k=3,4} m_k^o (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i b_j M}{\lambda_+ - \lambda_-} F(M_Y) , \quad (17)$$

$i, j = 1, 2, 3$ and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_+} \ln \frac{M_Y^2}{\lambda_+} - \frac{M_Y^2}{M_Y^2 - \lambda_-} \ln \frac{M_Y^2}{\lambda_-}$. Adding up all the one loop $SU(3)$ gauge boson contributions, we get the mass terms $\psi_L^o \mathcal{M}_1^o \psi_R^o + h.c.$,

$$\mathcal{M}_1^o = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ 0 & R_{22} & R_{23} & 0 \\ 0 & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (18)$$

$$R_{11} = \frac{1}{2}\mu_{22}F_1 + \frac{1}{2}\mu_{33}F_2 \quad R_{12} = \left(-\frac{1}{4}G_1 + \frac{1}{12}G_2\right)\mu_{12}$$

$$R_{22} = \frac{1}{4}\mu_{22}G_1 + \frac{1}{2}\mu_{33}F_3 + \frac{1}{12}\mu_{22}G_2 - G_m\mu_{22} \quad (19)$$

$$R_{33} = \frac{1}{2}\mu_{22}F_3 + \frac{1}{3}\mu_{33}G_2 \quad , \quad R_{13} = -\frac{1}{6}\mu_{13}G_2 - \mu_{13}G_m ,$$

$$R_{23} = -\frac{1}{6}\mu_{23}G_2 + \mu_{23}G_m \quad , \quad R_{32} = -\frac{1}{6}\mu_{32}G_2 + \mu_{32}G_m .$$

Here,

$$F_1 \equiv \frac{\alpha_2}{\pi}F(M_1) \quad , \quad F_2 \equiv \frac{\alpha_3}{\pi}F(M_2) \quad , \quad F_3 \equiv \frac{\alpha_3}{\pi}F(M_3)$$

$$G_1 \equiv \frac{\alpha_2}{\pi}G_{Z_1} \quad , \quad G_2 \equiv \frac{\alpha_3}{\pi}G_{Z_2}$$

$$G_m \equiv \frac{\sqrt{\alpha_2\alpha_3}}{\pi} \frac{\cos\phi \sin\phi [F(M_-) - F(M_+)]}{2\sqrt{3}}$$

$$G_{Z_1} = \cos\phi F(M_-) - \sin\phi F(M_+) \quad , \quad G_{Z_2} = \sin\phi F(M_-) + \cos\phi F(M_+) ,$$

where M_1 , M_2 , M_3 , M_-^2 , and M_+^2 are the horizontal boson masses defined in Eqs.(7,8) ,

$$\mu_{ij} = \frac{a_i b_j M}{\lambda_+ - \lambda_-} = \frac{a_i b_j}{a b} \sqrt{\lambda_-} c_\alpha c_\beta \quad , \quad \alpha_2 = \frac{g_{H_2}^2}{4\pi} \quad , \quad \alpha_3 = \frac{g_{H_3}^2}{4\pi} \quad (20)$$

and $c_\alpha \equiv \cos\alpha$, $c_\beta \equiv \cos\beta$, $s_\alpha \equiv \sin\alpha$, $s_\beta \equiv \sin\beta$, as defined in the Appendix, Eq.(A6). Therefore, up to one loop corrections we obtain the fermion masses

$$\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \psi_R^o = \bar{\chi}_L \mathcal{M} \chi_R , \quad (21)$$

with $\mathcal{M} \equiv [Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) + V_L^{oT} \mathcal{M}_1^o V_R^o]$.

Using V_L^o , V_R^o from Eqs.(A2) we get the mass matrix in Version I:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta m_{13} & s_\beta m_{13} \\ m_{21} & m_{22} & c_\beta m_{23} & s_\beta m_{23} \\ c_\alpha m_{31} & c_\alpha m_{32} & (-\sqrt{\lambda_-} + c_\alpha c_\beta m_{33}) & c_\alpha s_\beta m_{33} \\ s_\alpha m_{31} & s_\alpha m_{32} & s_\alpha c_\beta m_{33} & (\sqrt{\lambda_+} + s_\alpha s_\beta m_{33}) \end{pmatrix}, \quad (22)$$

where the mass entries m_{ij} ; $i, j = 1, 2, 3$ are written as:

$$m_{11} = \frac{1}{2} \frac{a_2}{a'} \Pi_1, \quad m_{12} = -\frac{1}{2} \frac{a_1 b_3}{a' b} (\Pi_2 - 6\mu_{22} G_m) \quad (23)$$

$$m_{21} = \frac{1}{2} \frac{a_1 a_3}{a' a} \Pi_1, \quad m_{31} = \frac{1}{2} \frac{a_1}{a} \Pi_1 \quad (24)$$

$$m_{13} = -\frac{1}{2} \frac{a_1 b_2}{a' b} [\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22} G_m] \quad (25)$$

$$m_{22} = \frac{1}{2} \frac{a_3 b_3}{a b} \left[\frac{a_2}{a'} (\Pi_2 - 6\mu_{22} G_m) + \frac{a' b_2}{a_3 b_3} (\Pi_3 + \Delta) \right] \quad (26)$$

$$m_{23} = \frac{1}{2} \frac{a_3 b_3}{a b} \left[\frac{a_2 b_2}{a' b_3} (\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22} G_m) - \frac{a'}{a_3} (\Pi_3 - \frac{b_2^2}{b_3^2} \Delta + 2\frac{b^2}{b_3^2} \mu_{33} G_m) \right] \quad (27)$$

$$m_{32} = \frac{1}{2} \frac{a_3 b_3}{a b} \left[\frac{a_2}{a_3} (\Pi_2 - 6\mu_{22} G_m) - \frac{b_2}{b_3} (\Pi_3 - \frac{a'^2}{a_3^2} \Delta - 2\frac{a^2}{a_3^2} \mu_{33} G_m) \right] \quad (28)$$

$$m_{33} = \frac{1}{2} \frac{a_3 b_3}{a b} \left[\frac{a_2 b_2}{a_3 b_3} (\Pi_2 - 2\mu_{22} G_m) + \Pi_3 + \frac{a'^2 b_2^2}{a_3^2 b_3^2} \Delta - \frac{1}{3} \frac{a^2 b^2}{a_3^2 b_3^2} \mu_{33} G_2 + 2(\frac{b_2^2}{b_3^2} + 2\frac{a_2^2}{a_3^2} - \frac{a'^2}{a_3^2}) \mu_{33} G_m \right] \quad (29)$$

$$\Pi_1 = \mu_{22} F_1 + \mu_{33} F_2, \quad \Pi_2 = \mu_{22} G_1 + \mu_{33} F_3$$

$$\Pi_3 = \mu_{22} F_3 + \mu_{33} G_2, \quad \Delta = \frac{1}{2} \mu_{33} (G_2 - G_1) \quad (30)$$

$$a' = \sqrt{a_1^2 + a_2^2}, \quad a = \sqrt{a'^2 + a_3^2}, \quad b = \sqrt{b_2^2 + b_3^2},$$

Notice that the m_{ij} mass terms up to one loop depend just on the ratios $\frac{a_i}{a_j} = \frac{v_i}{v_j} (\frac{V_i}{V_j})$ and $\frac{b_2}{b_3} = \frac{h_2 \Lambda_2}{h_3 \Lambda_3}$ coming from the tree level see-saw mass matrix \mathcal{M}^o .

For V_L^o , V_R^o from Eqs.(A3) we get the Version II:

$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & c_\beta M_{13} & s_\beta M_{13} \\ M_{21} & M_{22} & c_\beta M_{23} & s_\beta M_{23} \\ c_\alpha M_{31} & c_\alpha M_{32} & (-\sqrt{\lambda_-} + c_\alpha c_\beta M_{33}) & c_\alpha s_\beta M_{33} \\ s_\alpha M_{31} & s_\alpha M_{32} & s_\alpha c_\beta M_{33} & (\sqrt{\lambda_+} + s_\alpha s_\beta M_{33}) \end{pmatrix}, \quad (31)$$

where the mass terms $M_{ij}; i, j = 1, 2, 3$ may be obtained from those of m_{ij} as follows

$$\begin{aligned} M_{11} &= m_{22}, & M_{12} &= m_{21}, & M_{13} &= m_{23} \\ M_{21} &= -m_{12}, & M_{22} &= -m_{11}, & M_{23} &= -m_{13} \\ M_{31} &= m_{32}, & M_{32} &= m_{31}, & M_{33} &= m_{33} \end{aligned} \quad (32)$$

The diagonalization of \mathcal{M} , Eq.(22) or Eq.(31), gives the physical masses for u, d, e charged fermions. Neutrinos may also get left and right handed Majorana masses.

Using a new biunitary transformation $\chi_{L,R} = V_{L,R}^{(1)} \Psi_{L,R}; \bar{\chi}_L \mathcal{M} \chi_R = \bar{\Psi}_L V_L^{(1)T} \mathcal{M} V_R^{(1)} \Psi_R$, with $\Psi_{L,R}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$V_L^{(1)T} \mathcal{M} \mathcal{M}^T V_L^{(1)} = V_R^{(1)T} \mathcal{M}^T \mathcal{M} V_R^{(1)} = \text{Diag}(m_1^2, m_2^2, m_3^2, M_F^2), \quad (33)$$

$m_1^2 = m_e^2, m_2^2 = m_\mu^2, m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to massive fermion eigenfields read

$$\psi_L^o = V_L^o V_L^{(1)} \Psi_L \quad \text{and} \quad \psi_R^o = V_R^o V_R^{(1)} \Psi_R \quad (34)$$

C. Quark Mixing and non-unitary $(V_{CKM})_{4 \times 4}$

Recall that vector like quarks, Eq.(1), are $SU(2)_L$ weak singlets, and then they do not couple to W boson in the interaction basis. So, the interaction of ordinary quarks $f_{uL}^o{}^T = (u^o, c^o, t^o)_L$ and $f_{dL}^o{}^T = (d^o, s^o, b^o)_L$ to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \bar{f}_{uL}^o \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \bar{\Psi}_{uL} V_{uL}^{(1)T} [(V_{uL}^o)_{3 \times 4}]^T (V_{dL}^o)_{3 \times 4} V_{dL}^{(1)} \gamma_\mu \Psi_{dL} W^{+\mu}, \quad (35)$$

with g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4 \times 4} \equiv V_{uL}^{(1)T} [(V_{uL}^o)_{3 \times 4}]^T (V_{dL}^o)_{3 \times 4} V_{dL}^{(1)} \equiv V_{uL}^{(1)T} V_o V_{dL}^{(1)}. \quad (36)$$

For instance, for u and d quark masses \mathcal{M} in Eq.(31);

$$V_o = \begin{pmatrix} \frac{v_3 V_3}{vV} c\alpha + \frac{v' V'}{vV} & -\frac{v_3}{v} s\alpha & c_\alpha^d (\frac{v_3 V'}{vV} c\alpha - \frac{v' V_3}{vV}) & s_\alpha^d (\frac{v_3 V'}{vV} c\alpha - \frac{v' V_3}{vV}) \\ \frac{V_3}{V} s\alpha & c\alpha & \frac{V'}{V} c_\alpha^d s\alpha & \frac{V'}{V} s_\alpha^d s\alpha \\ c_\alpha^u (\frac{v' V_3}{vV} c\alpha - \frac{v_3 V'}{vV}) & -\frac{v'}{v} c_\alpha^u s\alpha & c_\alpha^d c_\alpha^u (\frac{v' V'}{vV} c\alpha + \frac{v_3 V_3}{vV}) & c_\alpha^u s_\alpha^d (\frac{v' V'}{vV} c\alpha + \frac{v_3 V_3}{vV}) \\ s_\alpha^u (\frac{v' V_3}{vV} c\alpha - \frac{v_3 V'}{vV}) & -\frac{v'}{v} s_\alpha^u s\alpha & c_\alpha^d s_\alpha^u (\frac{v' V'}{vV} c\alpha + \frac{v_3 V_3}{vV}) & s_\alpha^u s_\alpha^d (\frac{v' V'}{vV} c\alpha + \frac{v_3 V_3}{vV}) \end{pmatrix} \quad (37)$$

$$s_o = \frac{v_1}{v'} \frac{V_2}{V'} - \frac{v_2}{v'} \frac{V_1}{V'} \quad , \quad c_o = \frac{v_1}{v'} \frac{V_1}{V'} + \frac{v_2}{v'} \frac{V_2}{V'} \quad (38)$$

$$c_o^2 + s_o^2 = 1 \quad (39)$$

VI. PRELIMINARY NUMERICAL RESULTS FOR CHARGED LEPTONS

A. Charged leptons:

Using the strong hierarchy for quarks and charged leptons masses and the results in [27], we report here the magnitudes of lepton masses and mixing coming from a parameter space fit in this model.

In the approach $\Lambda_2 \approx \Lambda_3 = \Lambda$ or $\frac{M_2}{M_1} \approx \mathcal{O}(1)$:

$$M_1 \approx M_2 = M \quad , \quad M^2 = \frac{g_H^2 \Lambda^2}{2} \quad , \quad M_-^2 \approx \frac{2}{3} M^2 \quad , \quad M_+^2 \approx 2 M^2$$

Using the parameters from the $SU(3)$ flavour symmetry

$$\frac{\alpha_H}{\pi} = 0.06 \quad , \quad M = 10 \text{ TeV}$$

and the ones coming from the tree level seesaw

$$\sin \alpha = 0.1 \quad , \quad \sin \beta = 0.0081 \quad , \quad \sqrt{\lambda_-} = 2138.22 \text{ MeV} \quad , \quad \sqrt{\lambda_+} = 2.6 \text{ TeV}$$

$$\frac{V_1}{V_2} \simeq 0.1 \quad , \quad \frac{V_2}{\sqrt{V_1^2 + V_2^2}} \simeq 0.995037$$

$$\frac{\sqrt{V_1^2 + V_2^2}}{V_3} \simeq 0.344266 \quad , \quad \frac{h_3}{h_2} \simeq -0.218152$$

the mass matrix

$$M_e = \begin{pmatrix} 1.0558 & -9.6693 & -62.5208 & -0.5115 \\ -1.6717 & 102.263 & 7.7087 & 0.0630 \\ 39.19 & -3.3121 & -1744.59 & 3.2209 \\ 3.9387 & -0.3328 & 39.5616 & 2.6 \times 10^6 \end{pmatrix} \quad (40)$$

fit the charged lepton masses at the M_Z scale [28]:

$$(m_e, m_\mu, m_\tau) = (0.486, 102.7, 1746.17) \text{ MeV}$$

and a mass for the vector like electron of $M_E \approx 2.6 \text{ TeV}$ with mixing matrix

$$V_{eL}^{(1)} = \begin{pmatrix} 0.9950 & -0.0926 & -0.0358 & -1.9713 \times 10^{-7} \\ 0.0928 & 0.9956 & 0.0045 & 2.4300 \times 10^{-8} \\ -0.0352 & 0.0078 & -0.9993 & 1.2286 \times 10^{-6} \\ 2.3719 \times 10^{-7} & -5.2139 \times 10^{-8} & 1.2206 \times 10^{-6} & 1 \end{pmatrix} \quad (41)$$

VII. TREE LEVEL NEUTRINO MASSES

A. Tree level Dirac Neutrino masses

Here we present a preliminary study of neutrino masses. For neutrinos we may write the Dirac type gauge invariant couplings

$$h_D \bar{\Psi}_l^o \Phi^u N_R^o + h_2 \bar{\Psi}_\nu^o \eta_2 N_L^o + h_3 \bar{\Psi}_\nu^o \eta_3 N_L^o + M_D \bar{N}_L^o N_R^o + h.c \quad (42)$$

h , h_2 and h_3 are Yukawa couplings, and M_D a Dirac type invariant neutrino mass for the sterile neutrino $N_{L,R}^o$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi_{L,R}^{oT} = (\nu_e, \nu_\mu, \nu_\tau, N)_{L,R}^o$, the mass terms

$$\bar{\Psi}_{\nu L}^o \mathcal{M}_{\nu D}^o \Psi_{\nu R}^o + h.c.,$$

where

$$\mathcal{M}_D^{\nu(o)} = \begin{pmatrix} 0 & 0 & 0 & h_D v_1 \\ 0 & 0 & 0 & h_D v_2 \\ 0 & 0 & 0 & h_D v_3 \\ 0 & h_2 \Lambda_2 & h_3 \Lambda_3 & M_D \end{pmatrix} \quad (43)$$

B. Tree level Majorana masses:

Since $N_{L,R}^o$, Eq.(2), are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \bar{\Psi}_l^o \Phi (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c \quad (44)$$

and

$$h_{2R} \overline{(\Psi_\nu^o)^c} \eta_3^\dagger N_R^o + h_{3R} \overline{(\Psi_\nu^o)^c} \eta_2^\dagger N_R^o + m_R \bar{N}_R^o (N_R^o)^c + h.c., \quad (45)$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$\begin{aligned} & h_L \left[v_1 \overline{\nu_{eL}^o} + v_2 \overline{\nu_{\mu L}^o} + v_3 \overline{\nu_{\tau L}^o} \right] (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c \\ & + \left[h_{2R} \Lambda_2 \overline{(\nu_{\mu R}^o)^c} + h_{3R} \Lambda_3 \overline{(\nu_{\tau R}^o)^c} \right] N_R^o + m_R \bar{N}_R^o (N_R^o)^c + h.c., \end{aligned} \quad (46)$$

and the Generic Majorana mass matrix for neutrinos may be written as

$$(\bar{\Psi}_{\nu L}^o, \bar{\Psi}_{\nu L}^{oc}) \mathcal{M}_\nu \begin{pmatrix} \Psi_{\nu R}^{oc} \\ \Psi_{\nu R}^o \end{pmatrix}, \quad \Psi_{\nu L,R}^o = \begin{pmatrix} \nu_i^o \\ N^o \end{pmatrix}_{L,R}, \quad \Psi_{\nu L,R}^{oc} = \begin{pmatrix} \nu_i^{oc} \\ N^{oc} \end{pmatrix}_{L,R} \quad (47)$$

where

$$\mathcal{M}_\nu^{(o)} = \frac{1}{2} \begin{pmatrix} M_{\nu L}^o & M_{\nu D}^o \\ (M_{\nu D}^o)^T & M_{\nu R}^o \end{pmatrix} \quad (48)$$

and

$$M_{\nu L}^o = \begin{pmatrix} 0 & 0 & 0 & h_L v_1 \\ 0 & 0 & 0 & h_L v_2 \\ 0 & 0 & 0 & h_L v_3 \\ h_L v_1 & h_L v_2 & h_L v_3 & m_L \end{pmatrix}, \quad M_{\nu R}^o = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_{2R} \Lambda_2 \\ 0 & 0 & 0 & h_{3R} \Lambda_3 \\ 0 & h_{2R} \Lambda_2 & h_{3R} \Lambda_3 & m_R \end{pmatrix}, \quad (49)$$

VIII. ONE LOOP NEUTRINO MASSES

Diagonalization of the 8×8 Majorana mass matrix $\mathcal{M}_\nu^{(o)}$ in Eq.(48) yields four zero eigenvalues at tree level. Three of these massless neutrinos correspond to the active ordinary neutrinos. So, in this scenario light neutrinos may get very small masses from radiative corrections mediated by the $SU(3)$ heavy gauge bosons.

A. One loop Dirac Neutrino masses

Neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 2, as well as L-handed and R-handed Majorana masses from Fig. 3 and Fig. 4, respectively. The contribution from these diagrams read

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=1,2,3,4} m_k^o U_{ik}^o U_{jk}^o f(M_Y, m_k^o) = c_Y \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij} \quad , \quad \alpha_H \equiv \frac{g_H^2}{4\pi} \quad (50)$$

with

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=1,2,3,4} m_k^o U_{ik}^o U_{jk}^o f(M_Y, m_k^o), \quad (51)$$

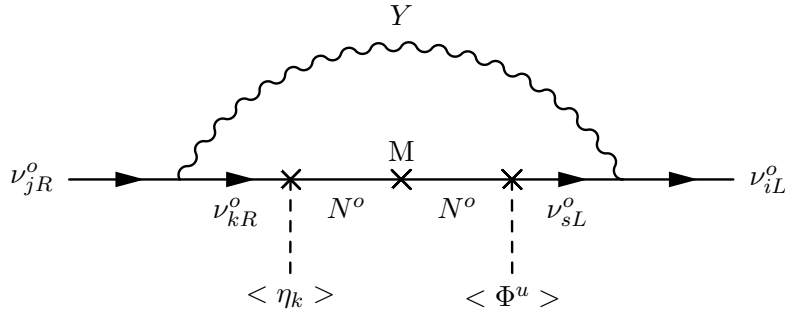


FIG. 2: Generic one loop diagram contribution to the Dirac mass term $m_{ij} \bar{\nu}_{iL}^o \nu_{jR}^o$. $M = M_D, m_L, m_R$

$$\mathcal{M}_{\nu D}^{(1)} = \begin{pmatrix} R_{\nu 15} & R_{\nu 16} & R_{\nu 17} & 0 \\ 0 & R_{\nu 26} & R_{\nu 27} & 0 \\ 0 & R_{\nu 36} & R_{\nu 37} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (52)$$

$$R_{\nu 15} = \frac{1}{2}m_{\nu}(M_1)_{26} + \frac{1}{2}m_{\nu}(M_2)_{37}, \quad R_{\nu 16} = -\frac{1}{4}m_{\nu}(M_{Z_1})_{16} + \frac{1}{12}m_{\nu}(M_{Z_2})_{16},$$

$$R_{\nu 26} = \frac{1}{4}m_{\nu}(M_{Z_1})_{26} + \frac{1}{2}m_{\nu}(M_3)_{37} + \frac{1}{12}m_{\nu}(M_{Z_2})_{26} - G_{\nu, m 26},$$

$$R_{\nu 37} = \frac{1}{2}m_{\nu}(M_3)_{26} + \frac{1}{3}m_{\nu}(M_{Z_2})_{37}, \quad R_{\nu 17} = -\frac{1}{6}m_{\nu}(M_{Z_2})_{17} - G_{\nu, m 17},$$

$$R_{\nu 27} = -\frac{1}{6}m_{\nu}(M_{Z_2})_{27} + G_{\nu, m 27}, \quad R_{\nu 36} = -\frac{1}{6}m_{\nu}(M_{Z_2})_{36} + G_{\nu, m 36},$$

$$m_{\nu}(M_{Z_1})_{ij} = \cos \phi m_{\nu}(M_-)_{ij} - \sin \phi m_{\nu}(M_+)_{ij}$$

$$m_{\nu}(M_{Z_2})_{ij} = \sin \phi m_{\nu}(M_-)_{ij} + \cos \phi m_{\nu}(M_+)_{ij}$$

$$G_{\nu, m ij} = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi} \frac{1}{2\sqrt{3}} \cos \phi \sin \phi [m_{\nu}(M_-)_{ij} - m_{\nu}(M_+)_{ij}] \quad (53)$$

B. One loop L-handed Majorana masses

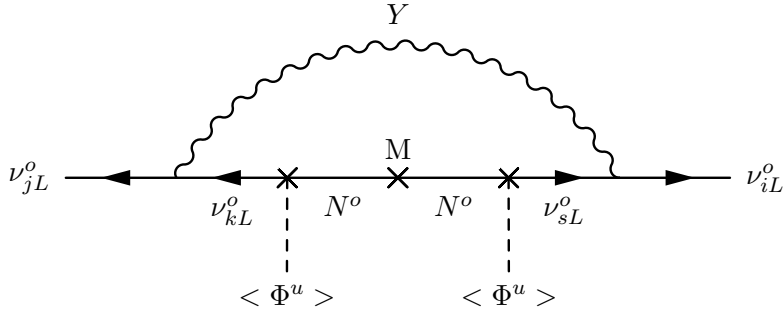


FIG. 3: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij} \bar{\nu}_{iL}^o (\nu_{jL}^o)^T$. $M = M_D, m_L, m_R$

$$M_{\nu L}^{(1)} = \begin{pmatrix} m_{\nu 11} & m_{\nu 12} & m_{\nu 13} & 0 \\ m_{\nu 12} & m_{\nu 22} & m_{\nu 23} & 0 \\ m_{\nu 13} & m_{\nu 23} & m_{\nu 33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (54)$$

$$\begin{aligned}
m_{\nu 11} &= \frac{1}{4} m - \nu(M_{Z_1})_{11} + \frac{1}{12} m_{\nu}(M_{Z_2})_{11} + G_{\nu, m 11} \\
m_{\nu 12} &= -\frac{1}{4} m_{\nu}(M_{Z_1})_{12} + \frac{1}{12} m_{\nu}(M_{Z_2})_{12} + \frac{1}{2} m_{\nu}(M_1)_{12} \\
m_{\nu 13} &= \frac{1}{2} m - \nu(M_2)_{13} - \frac{1}{6} m_{\nu}(M_{Z_2})_{13} - G_{\nu, m 13} \\
m_{\nu 22} &= \frac{1}{4} m_{\nu}(M_{Z_1})_{22} + \frac{1}{12} m_{\nu}(M_{Z_2})_{22} - G_{\nu, m 22} \\
m_{\nu 23} &= \frac{1}{2} m_{\nu}(M_3)_{23} - \frac{1}{6} m_{\nu}(M_{Z_2})_{23} + G_{\nu, m 23} \\
m_{\nu 33} &= \frac{1}{3} m_{\nu}(M_{Z_2})_{33}
\end{aligned} \tag{55}$$

C. One loop R-handed Majorana masses

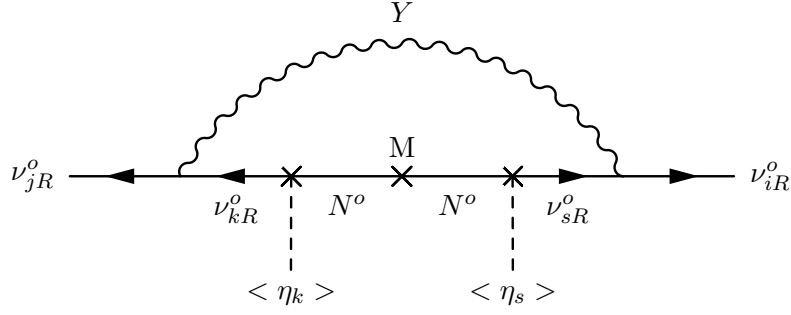


FIG. 4: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij} \bar{\nu}_{iR}^o (\nu_{jR}^o)^T$. $M = M_D, m_L, m_R$

$$M_{\nu R}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_{\nu 66} & m_{\nu 67} & 0 \\ 0 & m_{\nu 67} & m_{\nu 77} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{56}$$

$$\begin{aligned}
m_{\nu 66} &= \frac{1}{4} m_{\nu}(M_{Z_1})_{66} + \frac{1}{12} m_{\nu}(M_{Z_2})_{66} - G_{\nu, m 66} \\
m_{\nu 67} &= \frac{1}{2} m_{\nu}(M_3)_{67} - \frac{1}{6} m_{\nu}(M_{Z_2})_{67} + G_{\nu, m 67} \\
m_{\nu 77} &= \frac{1}{3} m_{\nu}(M_{Z_2})_{77}
\end{aligned} \tag{57}$$

IX. CONCLUSIONS

We have reported recent analysis on charged lepton and neutrino masses within a $SU(3)$ family symmetry model extension, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical spectrum for charged fermion masses and quark mixing.

We report in section VI a preliminary solution for ordinary charged lepton masses $(m_e, m_\mu, m_\tau) = (0.486, 102.7, 1746.17)$ MeV at the M_Z scale with a vector-like electron mass $M_E \approx 2.6$ TeV and horizontal gauge boson masses of the order of 10 TeV. A numerical fit on neutrino masses and mixing is in progress and results will be reported elsewhere.

It is worth to mention that due to the assigned of the ordinary fermions as triplets under $SU(3)$, each type of u, d, e and neutrinos couples just to one of the Φ^u or Φ^d scalars. So, u-quarks and neutrinos coupled to Φ^u while d-quarks and charged leptons couple to Φ^d . The scalar fields introduced to break the symmetries in the model: $\Phi, \Phi', \eta_1, \eta_2$ and η_3 , couple to ordinary fermions through the Eq.(11). Therefore, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

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Appendix A: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \quad (\text{A1})$$

Using a biunitary transformation $\psi_L^o = V_L^o \chi_L$ and $\psi_R^o = V_R^o \chi_R$ to diagonalize \mathcal{M}^o , the orthogonal matrices V_L^o and V_R^o may be written explicitly as the following two versions

Version I:

$$V_L^o = \begin{pmatrix} \frac{a_2}{a'} & \frac{a_1 a_3}{a' a} & \frac{a_1}{a} \cos \alpha & \frac{a_1}{a} \sin \alpha \\ -\frac{a_1}{a'} & \frac{a_2 a_3}{a' a} & \frac{a_2}{a} \cos \alpha & \frac{a_2}{a} \sin \alpha \\ 0 & -\frac{a'}{a} & \frac{a_3}{a} \cos \alpha & \frac{a_3}{a} \sin \alpha \\ 0 & 0 & -\sin \alpha & \cos \alpha \end{pmatrix}, \quad V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{b_3}{b} & \frac{b_2}{b} \cos \beta & \frac{b_2}{b} \sin \beta \\ 0 & -\frac{b_2}{b} & \frac{b_3}{b} \cos \beta & \frac{b_3}{b} \sin \beta \\ 0 & 0 & -\sin \beta & \cos \beta \end{pmatrix} \quad (\text{A2})$$

Version II:

$$V_L^o = \begin{pmatrix} \frac{a_1 a_3}{a' a} & -\frac{a_2}{a'} & \frac{a_1}{a} \cos \alpha & \frac{a_1}{a} \sin \alpha \\ \frac{a_2 a_3}{a' a} & \frac{a_1}{a'} & \frac{a_2}{a} \cos \alpha & \frac{a_2}{a} \sin \alpha \\ -\frac{a'}{a} & 0 & \frac{a_3}{a} \cos \alpha & \frac{a_3}{a} \sin \alpha \\ 0 & 0 & -\sin \alpha & \cos \alpha \end{pmatrix}, \quad V_R^o = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{b_3}{b} & 0 & \frac{b_2}{b} \cos \beta & \frac{b_2}{b} \sin \beta \\ -\frac{b_2}{b} & 0 & \frac{b_3}{b} \cos \beta & \frac{b_3}{b} \sin \beta \\ 0 & 0 & -\sin \beta & \cos \beta \end{pmatrix}, \quad (\text{A3})$$

$$\lambda_{\pm} = \frac{1}{2} \left(B \pm \sqrt{B^2 - 4D} \right) \quad (\text{A4})$$

are the nonzero eigenvalues of $\mathcal{M}^o \mathcal{M}^{oT}$ ($\mathcal{M}^{oT} \mathcal{M}^o$), and

$$B = a^2 + b^2 + c^2 = \lambda_- + \lambda_+ \quad , \quad D = a^2 b^2 = \lambda_- \lambda_+ , \quad (\text{A5})$$

$$\begin{aligned} \cos \alpha &= \sqrt{\frac{\lambda_+ - a^2}{\lambda_+ - \lambda_-}} \quad , \quad \sin \alpha = \sqrt{\frac{a^2 - \lambda_-}{\lambda_+ - \lambda_-}} , \\ \cos \beta &= \sqrt{\frac{\lambda_+ - b^2}{\lambda_+ - \lambda_-}} \quad , \quad \sin \beta = \sqrt{\frac{b^2 - \lambda_-}{\lambda_+ - \lambda_-}} . \end{aligned} \quad (\text{A6})$$